Electromagnetically induced torque on a large ring in the microwave range.

Olivier Emile, Christian Brousseau, Janine Emile, Ronan Niemiec, Kouroch Mahdjoubi, Bo Thide

Electromagnetically induced torque on a large ring in the microwave range

Olivier EMILE[1],* Christian BROUSSEAU[2], Janine EMILE[3],

Ronan NIEMIEC[1,2], Kouroch MADHJOUBI[2], and Bo THIDE[4]

[1] *URU 435 LPL, Université Rennes 1, 35042 Rennes cedex, France*

[2] *UMR CNRS 6164 IETR, Université Rennes 1, 35042 Rennes cedex, France*

[3] *UMR CNRS 6251 IPR, Université Rennes 1, 35042 Rennes cedex, France*

[4] *Swedish Institute of Space Physics, Ångström Laboratory,*

P.O.Box 537, SE-751 21 Uppsala, Sweden

(Dated: December 24, 2013)

Abstract

We report on the exchange of Orbital Angular Momentum between an electromagnetic wave and a 30 cm-diameter ring. Using a turnstile antenna in the GHz range, we induce a torque on a suspended copper strip of the order of 10^{-8} Nm. Rotations of a few degrees and accelerations up to 4. 10^{-4} $^\circ/s^2$ are observed. A linear dependence of the acceleration as a function of the applied power is found. There are many applications in the detection of angular momentum in electromagnetics, in acoustics and also in the magnetization of nanostructures.

PACS numbers: 41-20.Jb, 45-20.da, 84-40.Ba, 07.57.Kp,42.50.Tx

Angular momentum exchanges generally lead to mechanical torques. On the one hand in optics, in a pioneering experiment, Beth [1] demonstrates that the spin of electromagnetic (EM) waves can induce rotation of 2 cm-diameter birefringent plates. In his experiment he used parametric amplification to magnify the effect. This effect has since been experimentally confirmed in the macroscopic domain, both in optics [2, 3] and using microwaves [4, 5]. This transfer has also been experimentally evidenced in the microscopic [6] and submicroscopic scales [7, 8] using optics only. On the other hand, light can also carry Orbital Angular Momentum (OAM) [9–12]. OAM and Spin Angular Momentum (SAM) are usually considered as uncorrelated variables in the paraxial approximation [13–16]. Besides, OAM can exceed the maximum SAM of $\hbar$ per photon. This angular momentum has been used to induce rotation of microparticles [17–19], since there is no need for birefringence nor absorption, of Bose-Einstein condensates [20], and even of atoms [21]. However, in the microwave domain, no direct evidence of OAM transfer from an electromagnetic wave has ever been demonstrated. The aim of this letter is thus to explore the exchange between electromagnetic microwaves carrying OAM and a large suspended ring and to investigate the induced torque effect and its consequences.

The torsion pendulum has been used for over two centuries for fundamental measurements and is still the most sensitive tool for measuring weak torques either to test the gravitational inverse square law at very small length scales [22] or to study the dynamical response of threads [23]. It has also been used in photon angular momentum transfer to a suspended object [2–5]. We also chose a torsion pendulum to evidence the transfer of OAM. Moreover, since for a given power, OAM is proportional to the wavelength, we performed experiments in the microwave regime (see Fig.1). Actually, such a configuration has already been proposed few years ago by Vul'fson [24] but never realized. The antenna, also known as a turnstile antenna [25], is composed of two 17-cm-long, 2-mm-diameter copper dipole antennas. Such antenna is usually used to radiate circular polarized EM fields perpendicular to the antenna, however, in the plane of the antenna, it is supposed to radiate EM fields carrying $\ell = 1$ OAM [24].

According to the experimental radiation pattern, we chose to perform the experiment at a frequency $\nu = 870$ MHz. A -3dB coupler splits the monofrequency signal, generated from a frequency synthesizer in two, and induces a $\phi_1 - \phi_2 = \pi/2$ phase between the outputs. The signals are amplified by two 40 dB gain amplifiers and then sent to the antenna. The maximum total output is $P = 25$ W. The object to be rotated is a copper ring (radius $R = 15$ cm, height $H = 5$ cm, thickness $T = 165$ μm). Its inertial momentum is $J = 8.4\ 10^{-4}$ kgm^2. The suspension is a 2 m-long, 0.5 mm-diameter cotton thread. Its torsion constant is deduced from the 12 min period of the free oscillations of the pendulum and equals $C = 5.6\ 10^{-8}$ Nm/°. To avoid any spurious EM effect, the experiment is confined in an anechoic chamber. Special care is taken to isolate the set-up from any mechanical vibration. The copper ring is graduated and the rotation is recorded on a webcam connected to a computer.

Let us evaluate the electric and magnetic fields radiated by the turnstile antenna. We use the cylindrical coordinates (ρ, θ, z), The origin O being at the center of the turnstile antenna, $\mathbf{e_z}$ being perpendicular to it. However, the suspended ring doesn't lie in the far field and the whole components of the radiated fields have to be taken into account. The main results are presented here. The calculation is detailed in the Supplemental Material [26]. The complex potential vector, in the plane of the turnstile antenna, writes at a point M

$$\mathbf{A}(M,t) = -\frac{\mu_0}{4\pi}\frac{j\omega}{r}p_0 e^{j(kr-\omega t)}e^{j\theta}(\mathbf{e}_\rho + j\mathbf{e}_\theta), \tag{1}$$

where $r = OM$, ω is the pulsation of the current, p_0 is the dipole moment, μ_0 is the magnetic permeability and k is the wave vector. The complex magnetic field then writes

$$\mathbf{B}(M,t) = -\frac{\mu_0}{4\pi}\frac{j\omega}{r^2}p_0(1-jkr)e^{j(kr-\omega t)}e^{j(\theta-\pi/2)}\mathbf{e_z}. \tag{2}$$

The complex electric field writes

$$\mathbf{E}(M,t) = \frac{1}{4\pi\epsilon_0}\frac{p_0}{r^3}e^{j(kr-\omega t)}e^{j\theta}\left[2(1-jkr)\mathbf{e}_\rho - j(1-jkr-k^2r^2)\mathbf{e}_\theta\right], \tag{3}$$

ϵ_0 being the electric permittivity. One can note on this expression that, for a given distance r from the turnstile antenna, the modulus of the electric field is constant, its direction rotates around z, and, due to the $e^{j\theta}$ term in Eq.3, its phase varies from 0 to 2π in one turn. This is in agreement with simulations and experiments that have been performed on a turnstile antenna, in a different configuration, for radial slot antennas [27].

One could wonder whether this variation could be assigned to OAM. If one considers the far field only, one may jump to an erroneous conclusion. Indeed, in the far field, in the plane of the antenna, the polarization of the electric field is linear, there is no SAM. Considering the direction of the electric and magnetic fields, one may conclude that there is also no total angular momentum. Let us call Ω the solid angle subtended at the center of the antenna by the ring. It follows [13, 26] that the total angular momentum J equals

$$\mathbf{J} = \frac{\mu_0\omega}{(4\pi)^2}\frac{p_0^2}{r^2}\Omega(k^2r^2+1)\mathbf{e_z}, \tag{4}$$

Curiously enough, a near field component of the electric field contributes to the total angular momentum, even in the far field. The SAM S equals

$$\mathbf{S} = \frac{\mu_0\omega}{(4\pi)^2}\frac{p_0^2}{r^2}\Omega\mathbf{e_z}, \tag{5}$$

and the OAM L equals

$$\mathbf{L} = \mathbf{J} - \mathbf{S} = \frac{\mu_0\omega}{(4\pi)^2}p_0^2\Omega k^2\mathbf{e_z}, \tag{6}$$

From these equations, it turns out that the SAM has a small effect in the near field only whereas the OAM has the same and constant contribution in the near and far field. At the ring location ($r = R = 15$ cm), $L/S = k^2r^2 \approx 7.5$. Thus, in our case, the angular moment is mainly from OAM origin. From CST simulations [28], we have estimated that along the strip, even out of the plane of the turnstile antenna, the contribution of the OAM component to the total AM is 95% .

Fig.2 shows the experimental rotations of the pendulum versus time for three powers together with a parabola fitting of the curve. Clearly, the movement is a uniformly accelerated rotation. This is the first direct observation of the transfer of electromagnetic OAM, to a large object. For a longer time of observation, the restoring torque of the thread is no longer negligible. The rotation depends on the power, as expected from Eq.4. As the phase between the two dipole antennas is reversed, the sign of the OAM as well as the torque changes. Indeed we observed that the rotation direction of the pendulum is changed, as expected. Besides, as can be seen in Fig.2, the two plots of the rotation are nearly perfectly symmetric. Of course, when the two dipole antennas are in phase, no rotation is observed. We moved the copper ring (few centimeters) with respect to the turnstile antenna as well upwards and downwards, giving the same results.

Based on Fig.2, Fig.3 shows the angular acceleration induced by the EM field versus power. The experimental results evidence a linear dependence on the power, as expected, that holds over one order of magnitude. Note also that the linear coefficient is exactly reversed for the other direction of rotation. From Fig. 3, for a $P = 25$ W power, we find an acceleration of 3.6 10^{-4} $^\circ/s^2$ which corresponds to an OAM torque of 5.3 10^{-9} Nm. We can compare this value to the maximum angular momentum available carried by the EM fields per unit time, for a $\ell = 1$ wave and a power of 25 W, $\Gamma_{th} = \hbar N = \hbar(P/h\nu) = 4.6\ 10^{-9}$ Nm. Here N is the number of photons emitted per second. However, due to the solid angle subtended by the ring, roughly only one sixth of the total number of photons available interacts with the ring, assuming a uniform repartition. Each photon must thus be used several times via multiple reflections. Indeed one should consider the turnstile antenna and the copper ring as a whole system which is a much more complicated case. Simulations using CST indicate that the field radiated by the antenna induces alternating currents within the strip which magnitude equals 0.15 A. Actually these currents are not exactly alternating since there is a phase shift due to the $e^{j\theta}$ term in Eq.3. These currents are dissipated in the ring and thus induce a torque that is estimated to $\Gamma_{cal} = 7.0\ 10^{-9}$ Nm which is in reasonable agreement with the experimental value.

Conversely, due to the very high sensitivity of the torsion pendulum, we have been able to detect an applied torque due to the transfer of OAM as low as $6.3\ 10^{-10}$ Nm, corresponding to a power of 3 W (see Fig.2). Improving the suspension mechanics of the thread [23] and inserting the experimental set up in a vacuum chamber, a torque detection of 10^{-15} Nm seems within reach. The rotating object could be either a metallic strip in the case of spherical waves like ours, or a plate in the case of traveling waves. This would correspond to EM fields carrying OAM in the microwatt range or lower. These techniques could be an alternative way to detect OAM in the radio domain [29] or even in astronomy [30, 31]. Besides, like it has been recently demonstrated for propagating acoustic waves [32], such a detector measures the average angular momentum. Since the power of the beam is known, the topological charge could be then evaluated. Moreover, it could also determine the sign of the OAM.

To conclude, we have demonstrated the transfer of OAM from an electric field to a suspended object using a torsion pendulum. We have used a turnstile antenna to generate a microwave field that carries $\ell = 1$ angular momentum in a particular region of space where the contribution is mainly from OAM origin. We have isolated the accelerating regime and we were able to switch the direction of the rotation simply by reversing the phase of the antenna. It would be now stimulating to try to detect microwave field carrying higher order OAM ($\ell > 1$). Finally, since the first detection of spin angular momentum transfer was performed on electrons [33], it may be interesting to reproduce this experiment using electron waves carrying OAM [34, 35]. For example, such electron beams would induce a strong controllable magnetization to a nanowire inserted at the center of the vortex electron beam.

We would like to thank K. Wang, T. Teko, and A. Adibi for early interest, and D. Levalois and R. Legave for technical assistance. This work was supported by the University of Rennes 1 via a "défi émergent" action, the French Ministry of Defense (DGA) and the council of Brittany (Région Bretagne). We would also like to thank an anonymous referee for helpful comments.

* olivier.emile@univ-rennes1.fr

[1] R.A. Beth, Phys. Rev. **48**, 471 (1935).

[2] R.A. Beth, Phys. Rev. **50**, 115 (1936).

[3] G. Delannoy, O. Emile, and A. Le Floch, Appl. Phys. Lett. **86**, 081109 (2005).

[4] N. Carrara, Nature **164**, 882 (1949).

[5]] P.J. Allen, Am. J. Phys. **34**, 1185 (1966).

[6] M.E.J. Friese, T.A. Nieminen, N.R. Heckenberg, and H. Rubinsztein-Dunlop, Nature **86**, 348 (1998).

[7] B.E. Kane, Phys. Rev. B **82**, 115441 (2010).

[8] F. Pedaci, Z.X. Huang, M. van Oene, S. Barland, and N.H. Dekker, Nature Phys. **7**, 259 (2011).

[9] L. Allen, M.W. Beijersbergen, R.J.C. Spreeuw, and J.P. Woerdman, Phy. Rev. A **45**, 8185 (1992).

[10] L. Allen, S.M. Barnett, and M. Padgett, *Optical Angular Momentum*, (IOP Bristol, 2003).

[11] G. Molina-Terriza, J.P. Torres, and L. Torner, Nature Phys. **3**, 305 (2007).

[12] M. Padgett and R. Bowman, Nature Photonics **5**, 343 (2011).

[13] S.J. Van Enk and G. Nienhuis, J. Mod. Opt. **41**, 963 (1994).

[14] I. Bialynicki-Birula and Z. Bialynicka-Birula, J. Opt. **13**, 064014 (2011).

[15] V.V. Klimov, D. Bloch, M. Ducloy, and J.R. Rios Leite, Phys. Rev. A **85**, 053834 (2012).

[16] R.P. Cameron, S.M. Barnett, and A.M. Yao, New J. Phys. **14**, 053005 (2012).

[17] H. He, M.E.J. Friese, N.R. Heckenberg, and H. Rubinsztein-Dunlop, Phys. Rev. Lett. **75**, 826 (1995).

[18] S. Franke-Arnold, L. Allen, and M. Padgett, Laser and Photon. Rev. **2**, 299 (2008).

[19] D.B. Ruffner and D.G. Grier, Phys. Rev. Lett. **108**, 173602 (2012).

[20] M.F. Andersen, C. Ryu, P. Cladé, V. Natarajan, A. Vaziri, K. Helmerson, and W.D. Phillips, Phys. Rev. Lett. **97**, 170406 (2006).

[21] S. Barreiro and J.W.R. Tabosa, Phys. Rev. Lett. **90**, 133001 (2003).

[22] A.O. Sushkov, W.J. Kim, D.A.R. Dalvit, and S.K. Lamoreaux, Phys. Rev. Lett. **107** 171101 (2011).

[23] O. Emile, A. Le Floch, and A. Vollrath, Phys. Rev. Lett. **98**, 167402 (2007).

[24] K.S. Vul'fson, Sov. Phys. Usp. **30**, 724 (1988).

[25] J.D. Kraus *Antennas*, 2d ed. (McGraw-Hill New York, 1988).

[26] See Supplemental Material at the following URL for details of the calculations.

[27] K. Sudo, J. Hirokawa, M. Ando , Proc. 33rd Eur. Microwave Conf. **3**, 935 (2003).

[28] Computer Simulation Technology, CST, http://www.cst.com.

[29] B. Thidé, H. Then, J. Sjöholm, K. Palmer, J. Bergman, T.D. Carrozzi, Y.N. Istomin, N.H. Ibragimov, and R. Khanitova, Phys. Rev. Lett. **99**, 087701 (2007).

[30] N.M. Elias, Astron. Astrophys. **492**, 883 (2008).

[31] F. Tamburini, B. Thidé, G. Molina-Terriza, and G. Anzolin, Nature Phys. **7**, 195 (2011).

[32] C.E.M. Demore, Z.Y. Yang, A. Volovick, S. Cochran, M.P. MacDonald, and G.C. Spalding, Phys. Rev. Lett. **108**, 194301 (2012).

[33] A. Einstein and W.J. De Haas, Verh. Dtsch. Phys. Ges. **17**, 152 (1915).

[34] J. Verbeeck, H. Tian, and P. Schattschneider, Nature **467**, 301 (2010).

[35] B.J. McMorran, A. Agrawal, I.M. Anderson, A.A. Herzing, H.J. Lezec, J.J. McClelland, and J. Unguris, Science **331**, 192 (2011).

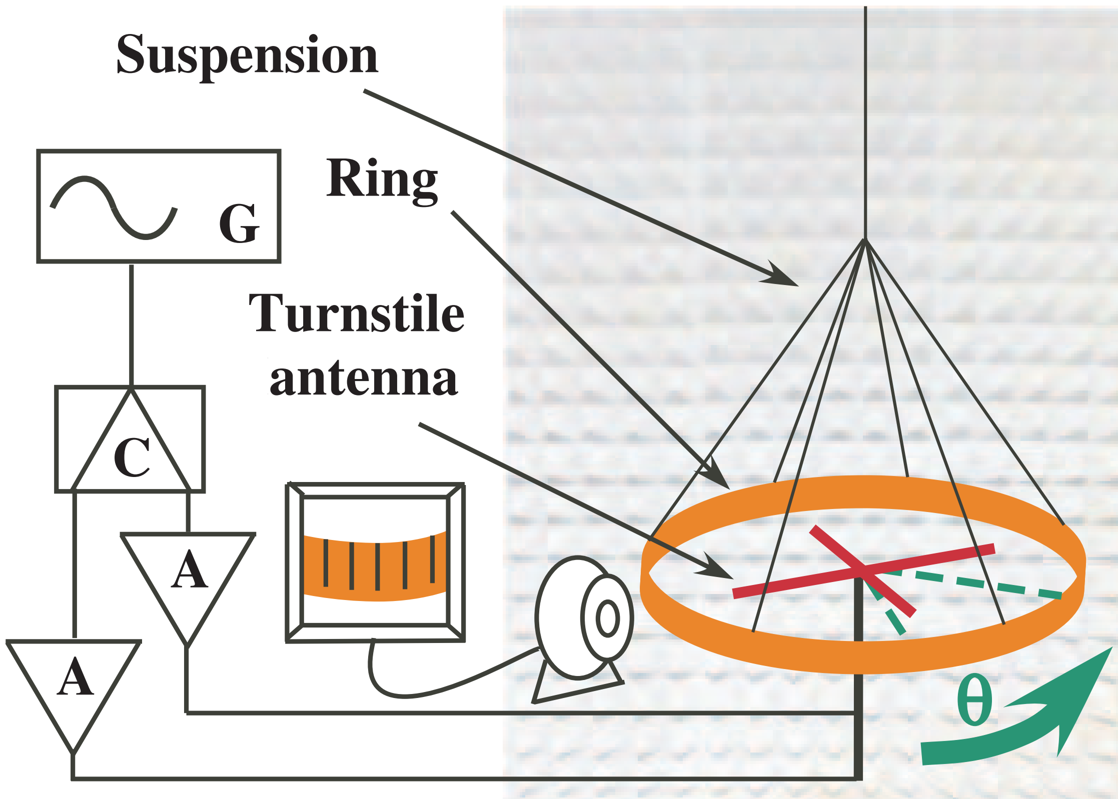

FIG. 1. Experimental set up. G: radio frequency generator, C: -3 dB coupler, A: 40 dB amplifier.

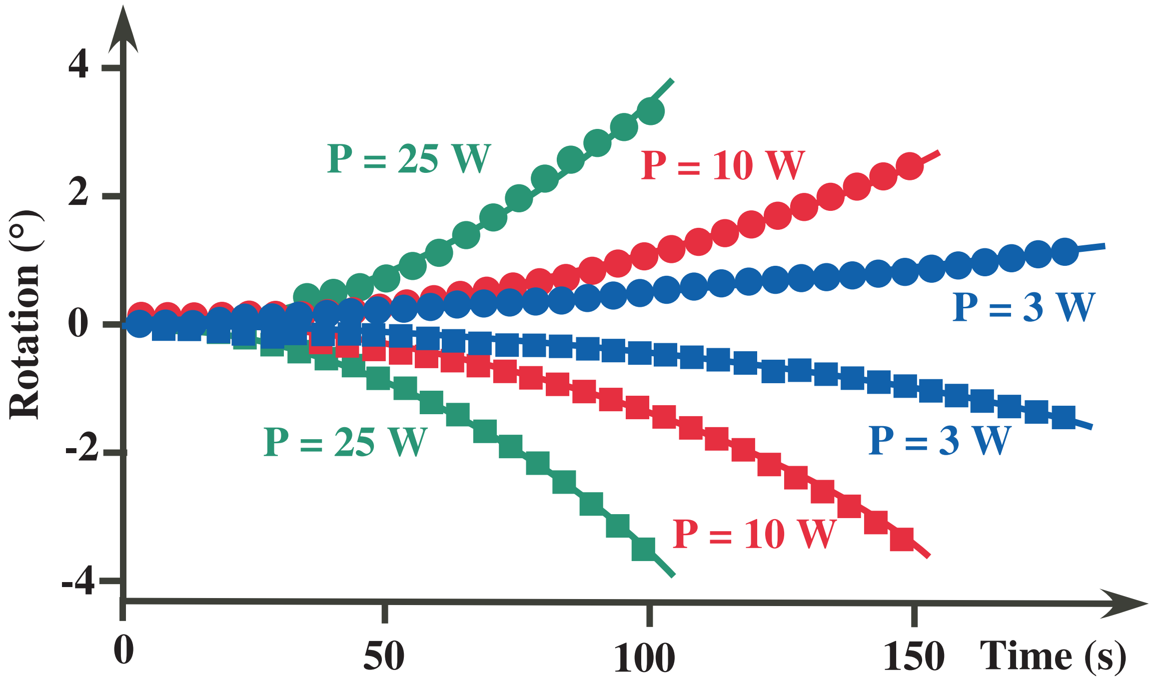

FIG. 2. Rotation versus time for a $\pi/2$ (circle) and $-\pi/2$ (square) phase between the two dipole antennas.

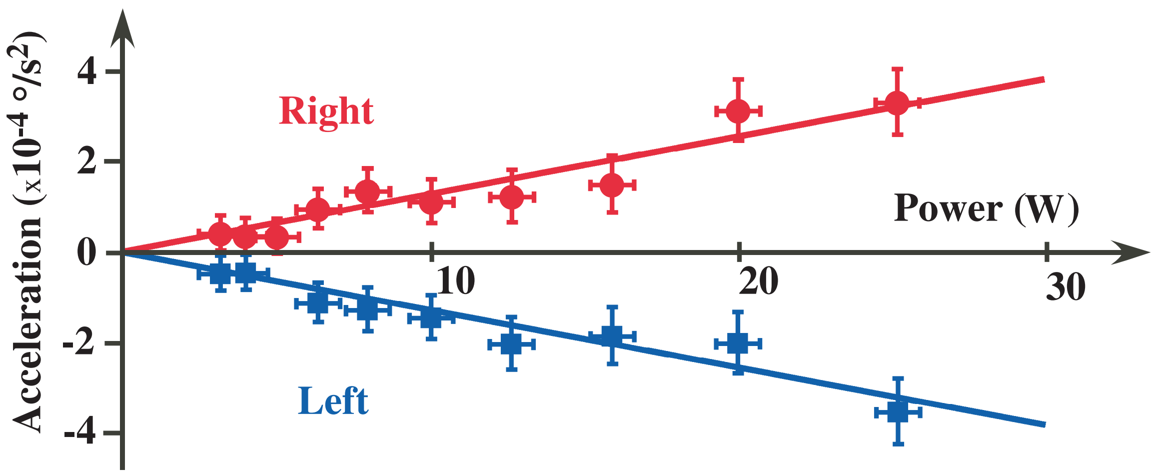

FIG. 3. Acceleration versus power for a $\pi/2$ (circle) and $-\pi/2$ (square) phase between the two dipole antennas.